

Who Said Neural Networks Aren't Linear?

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Neural networks are famously nonlinear. However, linearity is defined relative to a pair of vector spaces, $f: X \rightarrow Y$. Is it possible to identify a pair of non-standard vector spaces for which a conventionally nonlinear function is, in fact, linear? This paper introduces a method that makes such vector spaces explicit by construction. We find that if we sandwich a linear operator A between two invertible neural networks, $f(x) = g_Y^{-1}(A g_X(x))$ then the corresponding vector spaces X, Y are induced by newly defined addition and scaling actions derived from g_X and g_Y . We term this kind of architecture a Linearizer. This framework makes the entire arsenal of linear algebra, including SVD, pseudo-inverse, orthogonal projection and more, applicable to nonlinear mappings. Furthermore, we show that the composition of two Linearizers that share a neural network is also a Linearizer. We leverage this property and demonstrate that training diffusion models using our architecture makes the hundreds of sampling steps collapse into a single step. We further utilize our framework to enforce idempotency (i.e. $f(f(x))=f(x)$) on networks leading to a globally projective generative model and to demonstrate modular style transfer.